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New perspectives on Wireless Network Design

Strong, stable and robust 0-1 models by Power Discretization

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Abstract

This book presents the results that the Author has obtained in his multi-award winning work about the development of strong pure 0-1 Linear Programming formulations for the design of wireless networks. Specifically, the present publication considers the results contained in the Author's Ph.D. Thesis [DA10, DA12] (defended in January 2010 in front of the Ph.D. Evaluation Committee made up of Prof. A. Frangioni, Prof. V. Maniezzo and Prof. G. Zambelli) and the successive related developments and improvements, released through other publications (e.g., [BuDA12a, BuDA12b, DA11, DAMaSa11, DAMaSa12]).

1. Introduction

The wish of mankind to communicate from a distance without the use of physical connections dates back to very old times. In the work *Agamemnon*, Aeschylus (525-456 BC), the ancient Greek playwright and father of Tragedy, tells that the news of the fall of Troy was communicated through a system of eliographs to the royal palace in Argo, that was more than six hundred kilometers away [Fr99]. Nowadays, this wish has been finally satisfied: every day telecommunication systems allow billions of people to communicate crossing mountains, oceans and borders, actually realizing the idea of a Global Village introduced by Marshall McLuhan [ML64].

The key achievement for telecommunications was, without any doubt, the transmission of information through radio waves: the pioneering work of Guglielmo Marconi, that reached one of its more exciting climax with the transmission of signals across the ocean in 1901, opened the frontiers of communicating without the need for wires. The use of the appellative *wireless* has come in as the main term used to indicate networks that use radio waves to establish a connection between a receiver and a transmitter. Since the days of Marconi, wireless networks have shown a huge growth and now play an increasingly prominent role in different telecommunication systems. If we just consider wireless mobile services, the number of subscribers has grown from 11 million in 1990 to more than 4 billion in 2008 [ITU08].

In recent years, the evolution of wireless systems can be traced back to the desire to find a competitive alternative to traditional wireline-access technologies. Because of the deregulation in the field of telecommunications and the rapid growth of Internet, many companies have been spurred to look for a wireless solution to bypass consolidated service providers. This has led to the development of many wireless access systems that widely vary in their performances, supported applications, protocols, frequency spectrum used and many other aspects: television and radio programs are distributed through broadcasting networks (both terrestrial and satellite), mobile communication is ensured by cellular networks, internet is provided through broadband access networks. Moreover, a swarm of heterogeneous services are provided by ad-hoc wireless networks.

Wireless networks have grown very rapidly and very large during the last decades, generating a dramatic congestion of all radio resources. Though they rely on different technologies and standards, they still share a common feature: they all need to reach users scattered over a target area with a radio signal that must be strong enough to prevail against other unwanted signals. The perceived quality of service thus depends on several signals, wanted and unwanted, generated from possibly a large number of transmitting devices. Due to the large size of the current networks, to an extremely congested radio spectrum, to local and international constraints, establishing suitable emission powers for all the transmitters has become a very difficult systemic task, which calls for sophisticated optimization techniques. In the next Chapter we will formally introduce the problem of designing and planning a wireless network and show how Mathematical Programming can provide an invaluable support for this task.

2. The Wireless Network Design Problem

2.1. Introduction

In this section we introduce the modeling assumptions which provide the basis of the optimization model presented in Section (2.4).

A *wireless network* can be essentially described as a set of electronic devices that use electromagnetic waves to exchange information in a communication process. Every device is characterized by a set of parameters, generally divided into two categories: physical and radio-electrical. Physical parameters concern the location of the device (longitude, latitude, elevation, height of the device above the ground level, etc.), while radio-electrical parameters concern all the electronic aspects of the device (power emission, antenna diagram, frequency channel, transmission scheme, etc.).

Though all devices are in general able to transmit and receive signals, for modeling purposes the entire set of devices is usually partitioned into two disjoint sets: a set of *transmitters* B, which provide for the telecommunication services, and a set of *receivers* T, which make use of the services. Transmitters are assumed to be under the control of the network administrator. All the remaining devices belong instead to users. The main task of the administrator is to fix values of the transmitters parameters in order to provide for telecommunication services to the receivers, i.e. the devices possessed by the users.

Each transmitter $b \in B$ emits a radio signal with power $p_b \in [0, P_{\max}]$. Typically a receiver $t \in T$ receives radio signals from a subset $B(t) \subseteq B$ of transmitters. Since each transmitter in $B(t)$ is associated to a unique received signal, in what follows we will also refer to $B(t)$ as the set of signals received by t.

A receiver is said to be *covered* (or *served*) by the network if it receives the service within a minimum level of quality, defined on the basis of a service level agreement subscribed with the users.

The *Wireless Network Design Problem (WND)* corresponds with the point of view of a network administrator (public entity or private company) that wants to provide coverage to the users: the task is to establish suitable values for the parameters of the transmitters with the goal of maximizing a revenue functions associated with coverage (e.g., number of covered receivers, overall revenue from coverage).

We now proceed to present elements and concepts that are needed to introduce the analytical expression used to assess coverage.

2.2. Propagation models

Quality and reliability of a radio connection depend, besides on the radio-electrical parameters of the devices, also on propagation conditions experienced by signals. The planning phase therefore requires the adoption of a *propagation model* that is able to predict these conditions and to calculate the overall strength attenuation [Ra01]. This is not a simple task, as the easy computation of the *free space loss* must be adjusted by taking into account additional loss and degradation phenomena, which result from propagation in a real environment. The knowledge of landscape orography and human infrastructures is thus an essential requirement. Nowadays, morphological data of a geographical area are usually collected in large databases as *Digital Elevation Model (DEM)* files, the most widespread and used format. A DEM generally represents the surface of a region by means of a raster, whose elements specify relevant features (elevation and composition in terms of vegetation, buildings, etc.) of the corresponding elementary portion of territory. By considering these characteristics, a propagation model provides an attenuation coefficient that can be composed to other relevant contributions, such as antenna gain and connector loss, in order to obtain the total radio link budget.

All the gains and losses can be summarized into a *fading coefficient* $a_{tb} \in [0, 1]$ and the power $P_b(t)$ that a receiver $t \in T$ gets from a transmitter $b \in B$ is then proportional by a_{tb} to the power p_b emitted by transmitter b, namely:

$$P_b(t) = a_{tb} \cdot p_b. \tag{2.1}$$

2.3. Service coverage

As we have previously said, a receiver is *covered* (or *served*) by the network if the signal that carries the service is received with suitable quality. Among the received signals $B(t)$, receiver t can select a *reference signal* (or *server*), which is the one carrying the service. All the other signals are interfering (in digital broadcasting several signals can contribute to the overall wanted signal, but this case is not discussed here). A receiver t is regarded as served with reference signal $\beta \in B(t)$, if the following inequality in the emitted powers is satisfied:

$$\frac{a_{t\beta} \cdot p_\beta}{\sum_{b \in B \setminus \{\beta\}} a_{tb} \cdot p_b + N} \geq \delta. \tag{2.2}$$

The left member of the inequality coincide with the *Signal-to-Interference Ratio (SIR)*, that measures the ratio between the power received from the serving transmitter β and the sum of the powers associated to interfering signals [Ra01]. Note that a term N denoting the system noise is included among the interfering signals. Receiver β is regarded as covered if the SIR is higher than the value δ, generally denominated *SIR threshold*, in the right member of (2.2). The SIR threshold depends on the level of quality of service that is requested.

In order to simplify the discussion, we assume that all transmitters of the network operate on the same frequency (*Single Frequency Network* or simply *SFN*). This assumption is dropped in the experimental Section 3.1.3, where we describe the real-life application which motivated our developments.

By simple algebra operations, inequality (2.2) can be rearranged into the so-called *SIR inequality*:

$$a_{t\beta} \cdot p_\beta - \delta \sum_{b \in B(t) \setminus \{\beta\}} a_{tb} \cdot p_b \geq \delta \cdot N. \tag{2.3}$$

In order to simplify notation, in what follows we denote the product $\delta \cdot N$ as δ'.

Given a receiver $t \in T$, we have one inequality of type (2.3) for every potential server $\beta \in B(t)$ of t. As we do not know a priori which transmitter $\beta \in B(t)$ will be the server of t (this choice is part of the decision problem that we want to solve), we do not know which is the SIR inequality that we need to satisfy. However, we know that t is served if at least one SIR inequality corresponding to t is satisfied and so we are actually facing the following disjunctive constraint:

$$\bigvee_{\beta \in B(t)} \left(a_{t\beta} \cdot p_{\beta} - \delta \sum_{b \in B(t) \setminus \{\beta\}} a_{tb} \cdot p_b \geq \delta' \right). \tag{2.4}$$

If we introduce the following binary variable to denote coverage of a receiver $t \in T$ through a transmitter $b \in B(t)$:

$$x_{tb} \quad = \quad \begin{cases} 1 \text{ if receiver } t \in T \text{ is covered by transmitter } b \in B \\ 0 \text{ otherwise,} \end{cases}$$

disjunction (2.4) can be represented by the following family of linear constraints in the power variables:

$$a_{t\beta} \cdot p_{\beta} - \delta \sum_{b \in B(t) \setminus \{\beta\}} a_{tb} \cdot p_b + M \cdot (1 - x_{t\beta}) \geq \delta', \tag{2.5}$$

where M is a large positive constant generally known as *big-M*. Indeed, if $x_{t\beta} = 1$ then (2.5) reduces to (2.3); if instead $x_{t\beta} = 0$ and the big-M coefficient is sufficiently large (for example, we can set $M = \delta' + \delta \sum_{b \in B \setminus \{\beta\}} a_{tb} \cdot P_{\max}$), then (2.5) is satisfied for any feasible power vector p and becomes redundant.

2.4. A natural formulation for the WND

On the basis of the SIR inequality, it is straightforward to define a Mixed-Integer Linear Programming formulation for the WND. Such models are much exploited for wireless network optimization and belong to the class of the so called *big-M formulations* as they are based on the use of a big-M coefficient. We now present such a type of formulation, focusing attention on *downlink* transmission, i.e. signals propagating from transmitters to receivers.

The decision variables of the optimization model coincide with the variable mathematical entities in SIR inequality (2.3), introduced in the previous paragraphs:

1. continuous variables $p_b \in [0, P^{\max}]$ representing power emission of transmitter $b \in B$;

2. assignment variables $x_{tb} \in \{0, 1\}$ representing coverage of receiver $t \in T$ by transmitter $b \in B(t)$.

The aim is to maximize the overall revenue obtained by service coverage. If we introduce a parameter r_t to denote the revenue (e.g., population, number of customers, expected traffic demand) associated with receiver $t \in T$, the objective function is:

$$\max \quad \sum_{t \in T} \sum_{b \in B} r_t \cdot x_{tb}.$$

As each receiver is served at most by a single transmitter, for every receiver $t \in T$ we need to introduce the following packing constraint in the assignment variables:

$$\sum_{b \in B(t)} x_{tb} \leq 1.$$

By adding to these constraints the entire set of SIR inequalities (2.3) defined for each $t \in T$ and $b \in B(t)$, we can finally introduce a big-M formulation (BM), usually indicated by the appellative *"natural"* [DAMaSa09, Sa09], for the WND:

$$\begin{array}{llll}
\max & \sum_{t \in T} \sum_{b \in B(t)} r_t \cdot x_{tb} & & (BM) \\
& a_{t\beta} p_\beta - \delta \sum_{b \in B(t) \setminus \{\beta\}} a_{tb} p_b + M(1 - x_{t\beta}) \geq \delta' & t \in T, & (2.6) \\
& \sum_{b \in B(t)} x_{tb} \leq 1 & t \in T, & (2.7) \\
& p_b \leq P_{max} & b \in B, & \\
& p_b \geq 0 & b \in B, & \\
& x_{tb} \in \{0, 1\} & t \in T, b \in B(t). &
\end{array}$$

It is important to note that formulation (BM) does not take into account peculiar features of any specific wireless technology. However, technology-dependent versions can be obtained from the basic formulation by including suitable constraints or even new variables.

2.4.1. Drawbacks of the natural formulation

Natural formulations are widely used to model the WND, as proved by several studies in different application contexts, such as Radio and Video Broadcasting (e.g. [MaMaSa09, MaRoSm06]), GSM (e.g. [MaSc05]), UMTS (e.g. [AmCaMa01, EiEtal06, KaKeOl06, Na07]), WiMAX [DAMa08]. This is mainly due to the fact that they model the coverage problem in a very straightforward way, by directly including the SIR inequalities.

However, as we are dealing with a Mixed-Integer Linear Programming, it is widely known that "straightforward" modeling can lead to *weak* models that are very hard to solve [NeWo88, Wo98]. This is the case of the natural formulations: it is common experience that these formulations can be solved to optimality only when applied to small-sized instances [MaRoSm07]. In the case of large real-life instances, even finding feasible solutions can represent a difficult task, also for effective commercial MIP solvers such as ILOG Cplex [Cplex]. Moreover, solutions that are identified as feasible may actually contain coverage errors [DAMaSa09, KaKeOl06]. These difficulties are mainly determined by the following reasons:

- the general wireless network planning problem belongs to the class of *NP-hard* problems [MaRoSm06], and no polynomial time algorithm is known to the solution of (BM). This implies that solution time can grow very fast as the number of (binary) variables grows.

- the presence of the notorious big-M coefficient makes (BM) a *weak formulation*, that is the solution to its *linear programming relaxation*, obtained by removing the integrality stipulation on the variables, yields poor quality upper bounds [CoFi06]. This in turn drives standard MILP solution algorithms to generate larger search trees.

- the coefficient matrix of (BM) is (very) ill conditioned, because of the large range of feasible power values and overall attenuation coefficients of most real-life instances. Indeed, the ratio between the largest and the smallest coefficient in a SIR constraint (2.5) can be up to 10^{12}. This leads to numerical instability phenomena that heavily affects the effectiveness of LP-based solution algorithms.

In the following paragraph, we resume the main characteristics of the works about WND that have tried to overcome the above mentioned drawbacks and then we highlight what are the original research paths that we have beaten through this Ph.D. Thesis.

2.4.2. Overcoming the limits of natural formulations: past works

We have previously highlighted the three main issues associated to natural formulations: (i) huge hard-to-solve MILP problems; (ii) low quality